JAMA Guide to Statistics and Methods

Odds Ratios—Current Best Practice and Use

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Odds ratios frequently are used to present strength of association between risk factors and outcomes in the clinical literature. Odds and odds ratios are related to the probability of a binary outcome (an outcome that is either present or absent, such as mortality). The *odds* are the ratio of the probability that an outcome occurs to the probability that the outcome does not occur. For example, suppose that the probability of mortality is 0.3 in a group of patients. This can be expressed as the odds of dying: 0.3/(1 − 0.3) = 0.43. When the probability is small, odds are virtually identical to the probability. For example, for a probability of 0.05, the odds are 0.05/(1 − 0.05) = 0.052. This similarity does not exist when the value of a probability is large.

Probability and odds are different ways of expressing similar concepts. For example, when randomly selecting a card from a deck, the probability of selecting a spade is 13/52 = 25%. The odds of selecting a card with a spade are 25%/75% = 1:3. Clinicians usually are interested in knowing probabilities, whereas gamblers think in terms of odds. Odds are useful when wagering because they represent fair payouts. If one were to bet $1 on selecting a spade from a deck of cards, a payout of $3 is necessary to have an even chance of winning your money back. From the gambler's perspective, a payout smaller than $3 is unfavorable and greater than $3 is favorable.

Differences between 2 different groups having a binary outcome such as mortality can be compared using odds ratios, the ratio of 2 odds. Differences also can be compared using probabilities by calculating the *relative risk ratio*, which is the ratio of 2 probabilities. Odds ratios commonly are used to express strength of associations from logistic regression to predict a binary outcome.[1]

Why Report Odds Ratios From Logistic Regression?

Researchers often analyze a binary outcome using multivariable logistic regression. One potential limitation of logistic regression is that the results are not directly interpretable as either probabilities or relative risk ratios. However, the results from a logistic regression are converted easily into odds ratios because logistic regression estimates a parameter, known as the log odds, which is the natural logarithm of the odds ratio. For example, if a log odds estimated by logistic regression is 0.4 then the odds ratio can be derived by exponentiating the log odds (exp(0.4) = 1.5). It is the odds ratio that is usually reported in the medical literature. The odds ratio is always positive, although the estimated log odds can be positive or negative (log odds of −0.2 equals odds ratio of 0.82 = exp(−0.2)).

The odds ratio for a risk factor contributing to a clinical outcome can be interpreted as whether someone with the risk factor is more or less likely than someone without that risk factor to experience the outcome of interest. Logistic regression modeling allows the estimates for a risk factor of interest to be adjusted for other risk factors, such as age, smoking status, and diabetes. One nice feature of the logistic function is that an odds ratio for one covariate is constant for all values of the other covariates.

Another nice feature of odds ratios from a logistic regression is that it is easy to test the statistical strength of association. The standard test is whether the parameter (log odds) equals 0, which corresponds to a test of whether the odds ratio equals 1. Odds ratios typically are reported in a table with 95% CIs. If the 95% CI for an odds ratio does not include 1.0, then the odds ratio is considered to be statistically significant at the 5% level.

What Are the Limitations of Odds Ratios?

Several caveats must be considered when reporting results with odds ratios. First, the interpretation of odds ratios is framed in terms of odds, not in terms of probabilities. Odds ratios often are mistaken for relative risk ratios.[2,3] Although for rare outcomes odds ratios approximate relative risk ratios, when the outcomes are not rare, odds ratios always overestimate relative risk ratios, a problem that becomes more acute as the baseline prevalence of the outcome exceeds 10%. Odds ratios cannot be calculated directly from relative risk ratios. For example, an odds ratio for men of 2.0 could correspond to the situation in which the probability for some event is 1% for men and 0.5% for women. An odds ratio of 2.0 also could correspond to a probability of an event occurring 50% for men and 33% for women, or to a probability of 80% for men and 67% for women.

Second, and less well known, the magnitude of the odds ratio from a logistic regression is scaled by an arbitrary factor (equal to the square root of the variance of the unexplained part of binary outcome).[4] This arbitrary scaling factor changes when more or better explanatory variables are added to the logistic regression model because the added variables explain more of the total variation and reduce the unexplained variance. Therefore, adding more independent explanatory variables to the model will increase the odds ratio of the variable of interest (eg, treatment) due to dividing by a smaller scaling factor. In addition, the odds ratio also will change if the additional variables are not independent, but instead are correlated with the variable of interest; it is even possible for the odds ratio to decrease if the correlation is strong enough to outweigh the change due to the scaling factor.

Consequently, there is no unique odds ratio to be estimated, even from a single study. Different odds ratios from the same study cannot be compared when the statistical models that result in odds ratio estimates have different explanatory variables because each model has a different arbitrary scaling factor.[4-6] Nor can the magnitude of the odds ratio from one study be compared with the magnitude of the odds ratio from another study, because different samples and different model specifications will have different arbitrary scaling factors. A further implication is that the magnitudes of odds ratios of a given association in multiple studies cannot be synthesized in a meta-analysis.[4]

How Did the Authors Use Odds Ratios?

In a recent *JAMA* article, Tringale and colleagues[7] studied industry payments to physicians for consulting, ownership, royalties, and research as well as whether payments differed by physician specialty or sex. Industry payments were received by 50.8% of men across all specialties compared with 42.6% of women across all specialties. Converting these probabilities to odds, the odds that men receive industry payments is 1.03 (0.51/0.49), and the odds that women receive industry payments is 0.74 = (0.43/0.57).

The odds ratio for men compared with women is the ratio of the odds for men divided by the odds for women. In this case, the unadjusted odds ratio is 1.03/0.74 = 1.39. Therefore, the odds for men receiving industry payments are about 1.4 as large (40% higher) compared with women. Note that the ratio of the odds is different than the ratio of the probabilities because the probability is not close to 0. The unadjusted ratio of the probabilities for men and women (Tringale et al[7] report each probability, but not the ratio), the relative risk ratio, is 1.19 (0.51/0.43).

Greater odds that men may receive industry payments may be explained by their disproportionate representation in specialties more likely to receive industry payments. After controlling for specialty (and other factors), the estimated odds ratio was reduced from 1.39 to 1.28, with a 95% CI of 1.26 to 1.31, which did not include 1.0 and, therefore, is statistically significant. The odds ratio probably declined after adjusting for more variables because they were correlated with physicians' sex.

How Should the Findings Be Interpreted?

In exploring the association between physician sex and receiving industry payments, Tringale and colleagues[7] found that men are more likely to receive payments than women, even after controlling for confounders. The magnitude of the odds ratio, about 1.4, indicates the direction of the effect, but the magnitude of the number itself is hard to interpret. The estimated odds ratio is 1.4 when simultaneously accounting for specialty, spending region, sole proprietor status, sex, and the interaction between specialty and sex. A different odds ratio would be found if the model included a different set of explanatory variables. The 1.4 estimated odds ratio should not be compared with odds ratios estimated from other data sets with the same set of explanatory variables, or to odds ratios estimated from this same data set with a different set of explanatory variables.[4]

What Caveats Should the Reader Consider?

Odds ratios are one way, but not the only way, to present an association when the main outcome is binary. Tringale et al[7] also report absolute rate differences. The reader should understand odds ratios in the context of other information, such as the underlying probability. When the probabilities are small, odds ratios and relative risk ratios are nearly identical, but they can diverge widely for large probabilities. The magnitude of the odds ratio is hard to interpret because of the arbitrary scaling factor and cannot be compared with odds ratios from other studies. It is best to examine study results presented in several ways to better understand the true meaning of study findings.

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